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Onderwater, C. J. G.; LHCb Collaboration

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Observation of Several Sources of CP Violation in $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ DecaysR. Aaij *et al.*^{*}
(LHCb Collaboration)

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Observations are reported of different sources of CP violation from an amplitude analysis of $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ decays, based on a data sample corresponding to an integrated luminosity of 3 fb^{-1} of pp collisions recorded with the LHCb detector. A large CP asymmetry is observed in the decay amplitude involving the tensor $f_2(1270)$ resonance, and in addition significant CP violation is found in the $\pi^+ \pi^- S$ wave at low invariant mass. The presence of CP violation related to interference between the $\pi^+ \pi^- S$ wave and the P wave $B^+ \rightarrow \rho(770)^0 \pi^+$ amplitude is also established; this causes large local asymmetries but cancels when integrated over the phase space of the decay. The results provide both qualitative and quantitative new insights into CP -violation effects in hadronic B decays.

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Violation of symmetry under the combined charge-conjugation and parity-transformation operations, CP violation, gives rise to differences between matter and antimatter. Violation of CP symmetry can occur in the amplitudes that describe hadron decay, in neutral hadron mixing, or in the interference between mixing and decay (for a review, see, e.g., Ref. [1]). For charged mesons, only CP violation in decay is possible, where an asymmetry in particle and antiparticle decay rates can arise when two or more different amplitudes contribute to a transition. In particular, the phase of each complex amplitude can be decomposed into a weak phase, which changes sign under CP , and a strong phase, which is CP invariant. Differences in both the weak and strong phases of the contributing amplitudes are required for an asymmetry to occur.

In the standard model, weak phases arise from the elements of the Cabibbo-Kobayashi-Maskawa matrix [2,3] that are associated with quark-level transition amplitudes. Decays of B hadrons that do not contain any charm quarks in the final state, such as $B^+ \rightarrow \pi^+ \pi^+ \pi^-$, are of particular interest as both tree-level and loop-level amplitudes are expected to contribute with comparable magnitudes, so that large CP -violation effects are possible. Indeed, significant asymmetries have been observed in the two-body $B^0 \rightarrow K^+ \pi^-$ [4–6] and $B^0 \rightarrow \pi^+ \pi^-$ [4,6,7] decays. In two-body decays, nontrivial strong phases can arise from rescattering or other hadronic effects. In three-body or multibody decays, variation of the strong phase is

also expected due to the intermediate resonance structure, and hence amplitude analyses can provide additional sensitivity to CP -violation effects.

Analysis of the distribution of $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ decays (the inclusion of charge-conjugated processes is implied throughout this Letter, except where asymmetries are discussed) across the Dalitz plot [8,9], which provides a representation of the two-dimensional phase space for the decays, has been previously performed by the *BABAR* collaboration [10,11]. A model-independent analysis by the LHCb collaboration, with over an order of magnitude more signal decays and much better signal purity compared to the *BABAR* data sample, subsequently observed an intriguing pattern of CP violation in its phase space, notably in regions not associated to any known resonant structure [12,13]. The observed variation of the CP asymmetry across the Dalitz plot is expected to be related to the changes in strong phase associated with hadronic resonances, but, to date, it has not yet been explicitly described with an amplitude model. Many phenomenological studies [14–20] have provided possible interpretations of the asymmetries. Particular attention has been devoted to whether large CP -violation effects could arise from the interference between the broad low-mass spin-0 contributions and the spin-1 $\rho(770)^0$ resonance [21–24], from mixing between the $\rho(770)^0$ and $\omega(782)$ resonances [25–27], or from $\pi\pi \leftrightarrow K\bar{K}$ rescattering [21,23,24,28]. Further experimental studies are needed to clarify which of these sources are connected to the observed CP asymmetries.

In this Letter, results are reported on the amplitude structure of $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ decays, obtained by employing decay models that account for CP violation. The results are based on a data sample corresponding to 3 fb^{-1} of pp collisions at center-of-mass energies of 7 and 8 TeV, collected with the LHCb detector. A more detailed

^{*}Full author list given at the end of the article.

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description of the analysis is given in a companion paper [29]. The LHCb detector is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, described in detail in Refs. [30,31].

The selection of signal candidates closely follows the procedure used in the model-independent analysis of the same data sample [12], with minor enhancements. Events containing candidates are selected online by a trigger [32] that includes a hardware and software stage. The hardware stage requires either energy deposits in the calorimeters associated to signal particles or a trigger caused by other particles in the event. The software triggers require that the signal tracks come from a secondary vertex consistent with the decay of a b hadron. In the offline selection, two multivariate algorithms are used to separate the $B^+ \rightarrow \pi^+\pi^+\pi^-$ signal from background formed from random combinations of tracks, and from other B decays with misidentified final state particles, such as $B^+ \rightarrow K^+\pi^+\pi^-$. Candidates that originate from $B^+ \rightarrow \bar{D}^0\pi^+$ with subsequent $\bar{D}^0 \rightarrow \pi^+\pi^-$ or misidentified $K^+\pi^-$ decays are removed with a veto on both $\pi^+\pi^-$ invariant mass combinations.

After application of all selection requirements, the B^+ -candidate mass distribution is fitted to obtain signal and background yields. The fit function includes components for signal decays, combinatorial background and misidentified $B^+ \rightarrow K^+\pi^+\pi^-$ decays. The signal region in the B^+ candidate mass, $5.249 < m(\pi^+\pi^+\pi^-) < 5.317 \text{ GeV}/c^2$, which is used for the Dalitz-plot analysis, is estimated to contain a $20\,600 \pm 1600$ signal, a 4400 ± 1600 combinatorial background, and 143 ± 11 $B^+ \rightarrow K^+\pi^+\pi^-$ decays, where the uncertainties reflect the combination of statistical and systematic effects. The Dalitz-plot distributions of selected B^+ and B^- candidates are displayed in Fig. 1, where the phase space is folded by ordering the $\pi^+\pi^-$ pairs by their invariant mass, $m_{\text{low}} < m_{\text{high}}$.

Given the large number of broad overlapping resonances and decay-channel thresholds, it is particularly challenging to model the $B^+ \rightarrow \pi^+\pi^+\pi^-$ decay phenomenologically. Therefore, on top of the conventional “isobar” model using

a coherent sum of all nonzero spin resonances, three complementary approaches are used to describe the S -wave amplitude. The first continues in the isobar approach, comprising the coherent sum of a σ pole [33] together with a $\pi\pi \leftrightarrow K\bar{K}$ rescattering term [34]; the second uses the K -matrix formalism with parameters obtained from scattering data [35–37]; and the third implements a “quasi-model-independent” (QMI) approach, inspired by previous QMI analyses [38], where the dipion mass spectrum is divided into bins with independent magnitudes and phases that are free to vary in the amplitude fit.

The amplitude for B^+ and B^- signal decays is constructed as the sum over N resonant contributions and the S -wave component,

$$A^\pm(m_{13}^2, m_{23}^2) = \sum_{j=1}^N c_j^\pm F_j(m_{13}^2, m_{23}^2) + A_S^\pm(m_{13}^2, m_{23}^2), \quad (1)$$

where m_{13} and m_{23} denote the $\pi^+\pi^-$ invariant mass combinations. Bose symmetry is accounted for by enforcing the amplitude to be identical under interchange of the two like-sign pions, making the labeling of the two combinations arbitrary. The F_j term is the normalized dynamical amplitude of resonance j , represented by a mass line shape multiplied by the spin-dependent angular distribution using the Zemach tensor formalism [39,40] and Blatt-Weisskopf barrier factors [41]. The complex coefficients, c_j^\pm , give the relative contribution of each resonance, and A_S^\pm is the S -wave amplitude (isobar, K matrix, or QMI). The amplitude models account for CP -violating differences between the distributions of B^+ and B^- decays by allowing the c_j^\pm coefficients, and relevant parameters in A_S^\pm , to take different values in the two cases. A likelihood function is constructed from the squared magnitude of the signal amplitude, accounting for efficiency effects and normalization, and including background contributions modeled from data sidebands and simulation. The signal parameters are evaluated in the fit by minimizing the

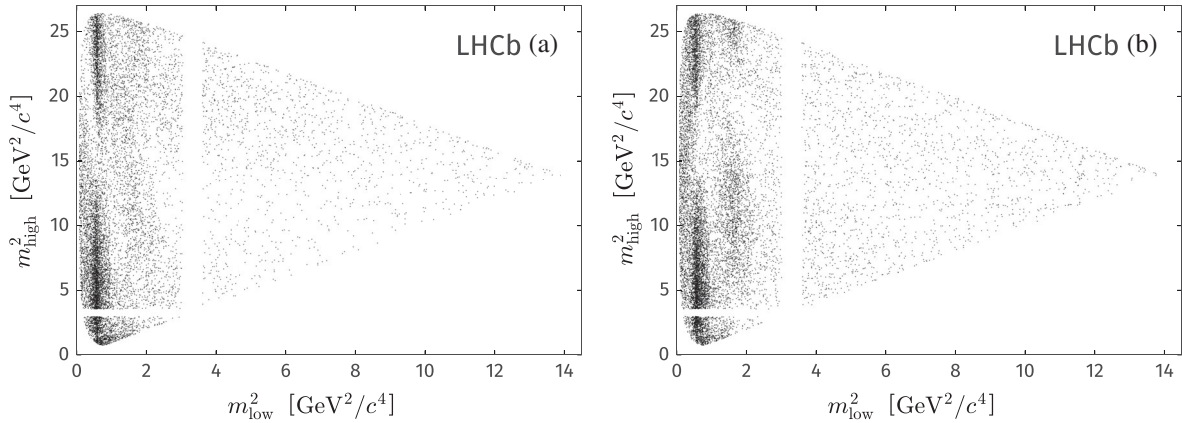


FIG. 1. Dalitz-plot distributions for (a) B^+ and (b) B^- candidate decays to $\pi^+\pi^+\pi^-$. Depleted regions are due to the \bar{D}^0 veto.

negative logarithm of the total likelihood, calculated for all candidates in the signal region. The LAURA++ package [42] is used for the isobar and K -matrix approaches, while a GPU-accelerated version of the MINT2 fitter [43] is used for the QMI approach.

With the exception of the S wave, the included components are identical in each approach and consist of the $\rho(770)^0$ and $\omega(782)$ resonances described by a coherent ρ - ω mixing model [44], and the $f_2(1270)$, $\rho(1450)^0$, and $\rho_3(1690)^0$ resonances. These latter three resonances are all described by relativistic Breit-Wigner line shapes. The choice of which resonances to include is made starting from the model obtained in the BABAR analysis [11], with additional contributions included if they cause a significant improvement in the fit to data.

In each approach, model coefficients for B^+ and B^- decays are obtained simultaneously. The amplitude coefficients extracted from the fit, $c_j^\pm = (x \pm \delta x) + i(y \pm \delta y)$, where positive (negative) signs are used for B^+ (B^-) decays, are defined such that CP violation is permitted. For the dominant ρ - ω mixing component, the magnitude of the coefficient in the B^+ amplitude is fixed to unity to set the scale, while both B^+ and B^- coefficients are aligned to the real axis as the absolute phase carries no physical meaning.

Good overall agreement between the data and the model is obtained for all three S -wave approaches, with some

localized discrepancies that are discussed below. Moreover, the values for the CP -averaged fit fractions and quasi-two-body CP asymmetries (rate asymmetries between a quasi-two-body decay and its CP conjugate), derived from the fit coefficients and given in Table I, show good agreement between the three approaches.

Projections of the data and the fit models are shown in regions of the data with $m(\pi^+\pi^-) < 1$ GeV/ c^2 in Fig. 2. The $\rho(770)^0$ resonance is found to be the dominant component in all models, with a fit fraction of around 55% and a quasi-two-body CP asymmetry that is consistent with zero. The effect of ρ - ω mixing is very clear in the data [Fig. 2(b)] and is well described by the models. Contrary to some theoretical predictions [25–27], there is no evident CP -violation effect associated with ρ - ω mixing. However, a clear CP asymmetry is seen at values of $m(\pi^+\pi^-)$ below the $\rho(770)^0$ resonance, where only the S -wave amplitude contributes significantly [Fig. 2(a)]. A detailed inspection of the behavior of the S wave, given in Ref. [29], shows that this CP asymmetry remains approximately constant up to the inelastic threshold $2m_K$, where it appears to change sign; this is seen in all three approaches to the S wave description. Estimates of the significance of this CP -violation effect give values in excess of ten Gaussian standard deviations (σ) in all the S -wave models. These estimates are obtained from the change in negative log-likelihood between, for each S -wave approach,

TABLE I. Results for CP -conserving fit fractions, quasi-two-body CP asymmetries, and phases for each component relative to the combined $\rho(770)^0$ - $\omega(782)$ model, given for each S -wave approach. The $\rho(770)^0$ and $\omega(782)$ values are extracted from the combined $\rho(770)^0$ - $\omega(782)$ mixing model. The first uncertainty is statistical while the second is systematic.

Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase ($^\circ$)	B^- phase ($^\circ$)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$

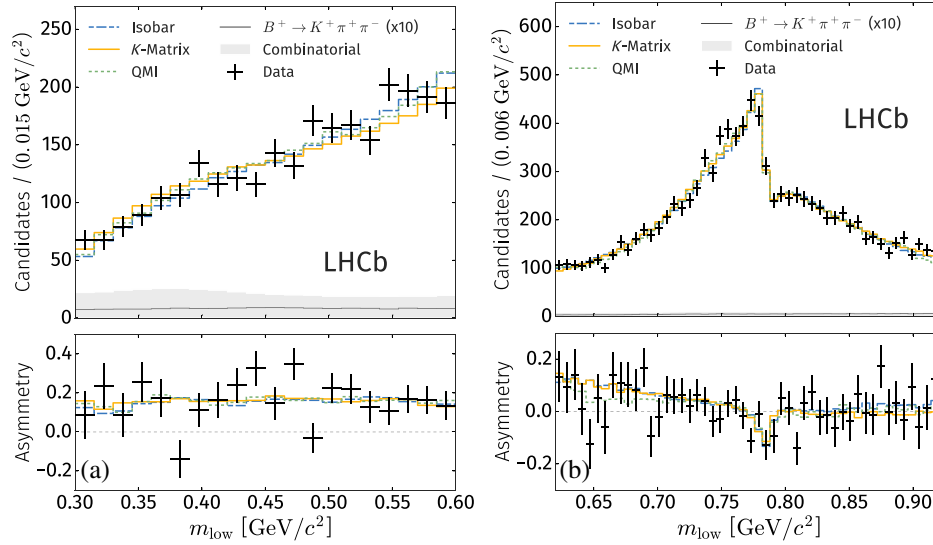


FIG. 2. Projections of data and fits (top) on m_{low} in (a) the low $m(\pi^+\pi^-)$ region and (b) the ρ - ω region, with (bottom) the corresponding CP asymmetries in these ranges.

the baseline fit and alternative fits where no such CP violation is allowed.

An additional source of CP violation, associated principally with the interference between S and P waves, is clearly visible when inspecting the $\cos\theta_{\text{hel}}$ distributions separately in regions above and below the $\rho(770)^0$ peak [Figs. 3(a) and 3(b)]. Here, θ_{hel} is the angle, evaluated in the $\pi^+\pi^-$ rest frame, between the pion with opposite charge to

the B and the third pion from the B decay. These asymmetries are modeled well in all three approaches to the S -wave description. Evaluation of the significance of CP violation in the interference between S and P waves gives values in excess of 25σ in all the S -wave models.

At higher $m(\pi^+\pi^-)$ values, the $f_2(1270)$ component is found to have a CP -averaged fit fraction of around 9% and a very large quasi-two-body CP asymmetry of around 40%, as can be seen in Fig. 4 and Table I. This is the first observation of CP violation in any process involving a

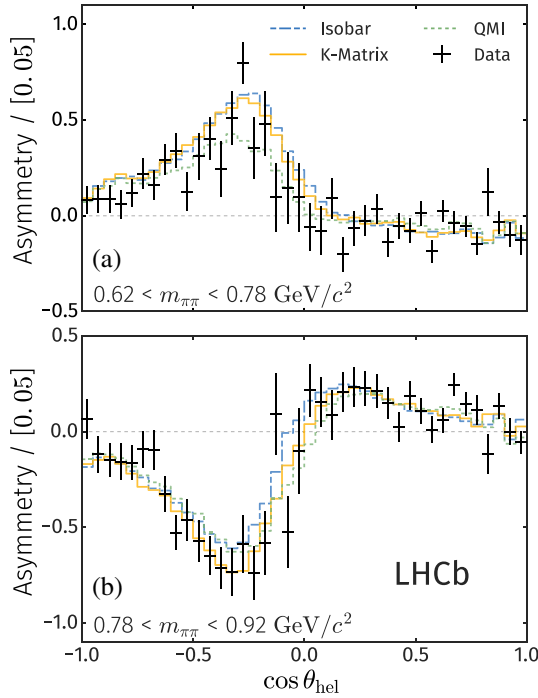


FIG. 3. Projections of the CP asymmetry for data and fits as a function of $\cos\theta_{\text{hel}}$ in the regions (a) below and (b) above the $\rho(770)^0$ resonance pole.

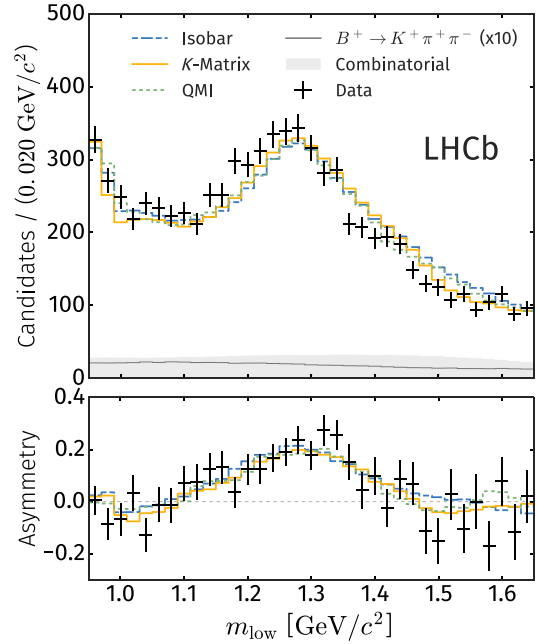


FIG. 4. Projections of data and fits (top) on m_{low} in the $f_2(1270)$ mass region, with (bottom) the corresponding CP asymmetry.

tensor resonance. The central value of the CP asymmetry is consistent with some theoretical predictions [19,45,46] that, however, have large uncertainties. The significance of CP violation in the complex amplitude coefficients of the $f_2(1270)$ component is in excess of 10σ . This conclusion holds in all the S -wave models and is robust against variations of the models performed to evaluate systematic uncertainties.

The parameters associated to the $\rho(1450)^0$ and $\rho_3(1690)^0$ resonances agree less well, but are nevertheless broadly consistent, between the different models. The small $\rho_3(1690)^0$ contribution exhibits a large quasi-two-body CP asymmetry; however this result is subject to significant systematic uncertainties, particularly due to ambiguities in the amplitude model, and therefore is not statistically significant.

The main sources of experimental systematic uncertainty are related to the signal, combinatorial and peaking background parametrization in the B^+ invariant-mass fit, and the description of the efficiency variation across the Dalitz plot. Also considered, and found to be numerically larger for most results, are systematic uncertainties related to the physical amplitude models. These comprise the variation of masses and widths, according to the world averages [47], of established resonances, in addition to the inclusion of more speculative resonant structures. A small contribution from the $\rho(1700)^0$ resonance is expected by some theory predictions [48] and is considered a source of systematic uncertainty since the inclusion of this term did not significantly improve the models' agreement with data.

A clear discrepancy between all three modeling approaches and the data can be observed in the $f_2(1270)$ region (Fig. 4). This discrepancy can be resolved by freeing the $f_2(1270)$ mass parameter in the fit; however, the values obtained are significantly different from the world-average value. The discrepancy could arise from interference with an additional spin-2 resonance in this region, but all well-established states are either too high in mass or too narrow in width to be likely to cause a significant effect. The inclusion of a second spin-2 component in this region, with free mass and width parameters, results in values of the $f_2(1270)$ mass consistent with the world average, where parameters of the additional state are broadly consistent with those of the speculative $f_2(1430)$ resonance; however the values obtained for the mass and width of the additional state are inconsistent between fits with different approaches to the S -wave description. Subsequent analysis of larger data samples will be required to obtain a more detailed understanding of the $\pi\pi D$ wave in $B^+ \rightarrow \pi^+\pi^+\pi^-$ decays. Variation of the $f_2(1270)$ mass with respect to the world-average value, along with the addition of a second spin-2 resonance in this region, are taken into account in the systematic uncertainties.

In summary, an amplitude analysis of the $B^+ \rightarrow \pi^+\pi^+\pi^-$ decay is performed with data corresponding to 3 fb^{-1} of

LHCb Run 1 data, using three complementary approaches to describe the large S -wave contribution to this decay. Good agreement is found between all three models and the data. In all cases, significant CP violation is observed in the decay amplitudes associated with the $f_2(1270)$ resonance and with the $\pi^+\pi^-S$ wave at low invariant mass, in addition to CP violation characteristic of interference between the spin-1 $\rho(770)^0$ resonance and the spin-0 S -wave contribution. Violation of CP symmetry is previously unobserved in these processes and, in particular, this is the first observation of CP violation in the interference between two quasi-two-body decays. As such, these results provide significant new insight into how CP violation manifests in multibody B -hadron decays, and motivate further study into the processes that govern CP violation at low $\pi\pi$ invariant mass.

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Cavallero,²² R. Cenci,^{27,i} M. G. Chapman,⁵¹ M. Charles,^{11,45} Ph. Charpentier,⁴⁵ G. Chatzikonstantinidis,⁵⁰ M. Chefdeville,⁷ V. Chekalina,⁴⁰ C. Chen,³ S. Chen,²⁵ S.-G. Chitic,⁴⁵ V. Chobanova,⁴⁴ M. Chruszcz,⁴⁵ A. Chubykin,³⁶ P. Ciambone,²¹ X. Cid Vidal,⁴⁴ G. Ciezarek,⁴⁵ F. Cindolo,¹⁸ P. E. L. Clarke,⁵⁵ M. Clemencic,⁴⁵ H. V. Cliff,⁵² J. Closier,⁴⁵ J. L. Cobbledick,⁵⁹ V. Coco,⁴⁵ J. A. B. Coelho,¹⁰ J. Cogan,⁹ E. Cogneras,⁸ L. Cojocariu,³⁵ P. Collins,⁴⁵ T. Colombo,⁴⁵ A. Comerma-Montells,¹⁵ A. Contu,²⁵ G. Coombs,⁴⁵ S. Coquereau,⁴³ G. Corti,⁴⁵ C. M. Costa Sobral,⁵³ B. Couturier,⁴⁵ G. A. Cowan,⁵⁵ D. C. Craik,⁶¹ A. Crocombe,⁵³ M. Cruz Torres,¹ R. Currie,⁵⁵ C. L. Da Silva,⁶⁴ E. Dall'Occo,³⁰ J. Dalseno,^{44,51} C. D'Ambrosio,⁴⁵ A. Danilina,³⁷ P. d'Argent,¹⁵ A. Davis,⁵⁹ O. De Aguiar Francisco,⁴⁵ K. De Bruyn,⁴⁵ S. De Capua,⁵⁹ M. De Cian,⁴⁶ J. M. De Miranda,¹ L. De Paula,² M. De Serio,^{17,j} P. De Simone,²¹ J. A. de Vries,³⁰ C. T. Dean,⁵⁶ W. Dean,⁷⁸ D. Decamp,⁷ L. Del Buono,¹¹ B. Delaney,⁵² H.-P. Dembinski,¹⁴ M. Demmer,¹³ A. Dendek,³³ D. Derkach,⁷⁵ O. Deschamps,⁸ F. Desse,¹⁰ F. Dettori,²⁵ B. Dey,⁶ A. Di Canto,⁴⁵ P. Di Nezza,²¹ S. Didenko,⁷⁴ H. Dijkstra,⁴⁵ F. Dordei,²⁵ M. Dorigo,^{27,k} A. C. dos Reis,¹ A. Dosil Suárez,⁴⁴ L. Douglas,⁵⁶ A. Dovbnya,⁴⁸ K. Dreimanis,⁵⁷ L. Dufour,⁴⁵ G. Dujany,¹¹ P. Durante,⁴⁵ J. M. Durham,⁶⁴ D. Dutta,⁵⁹ R. Dzhelyadin,^{42,a} M. Dziewiecki,¹⁵ A. Dziurda,³² A. Dzyuba,³⁶ S. Easo,⁵⁴ U. Egede,⁵⁸ V. Egorychev,³⁷ S. Eidelman,^{41,e} S. Eisenhardt,⁵⁵ U. Eitschberger,¹³ R. Ekelhof,¹³ S. Ek-In,⁴⁶ L. Eklund,⁵⁶ S. Ely,⁶⁵ A. Ene,³⁵ S. Escher,¹² S. Esen,³⁰ T. Evans,⁶² A. Falabella,¹⁸ C. Färber,⁴⁵ N. Farley,⁵⁰ S. Farry,⁵⁷ D. Fazzini,¹⁰ M. Féo,⁴⁵ P. Fernandez Declara,⁴⁵ A. Fernandez Prieto,⁴⁴ F. Ferrari,^{18,c} L. Ferreira Lopes,⁴⁶ F. Ferreira Rodrigues,² S. Ferreres Sole,³⁰ M. Ferro-Luzzi,⁴⁵ S. Filippov,³⁹ R. A. Fini,¹⁷ M. Fiorini,^{19,f} M. Firlej,³³ C. Fitzpatrick,⁴⁵ T. Fiutowski,³³ F. Fleuret,^{10,b} M. Fontana,⁴⁵ F. Fontanelli,^{22,l} R. 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 S. Karodia,⁵⁶ N. Kazeev,⁷⁵ M. Kecke,¹⁵ F. Keizer,⁵² M. Kelsey,⁶⁵ M. Kenzie,⁵² T. Ketel,³¹ B. Khanji,⁴⁵ A. Kharisova,⁷⁶
 C. Khurewathanakul,⁴⁶ K. E. Kim,⁶⁵ T. Kim,¹² V. S. Kirsebom,⁴⁶ S. Klaver,²¹ K. Klimaszewski,³⁴ S. Koliev,⁴⁹ M. Kolpin,¹⁵
 A. Kondybayeva,⁷⁴ A. Konoplyannikov,³⁷ P. Kopciwicz,³³ R. Kopečna,¹⁵ P. Koppenburg,³⁰ I. Kostiuk,^{30,49} O. Kot,⁴⁹
 S. Kotriakhova,³⁶ M. Kozeiha,⁸ L. Kravchuk,³⁹ M. Kreps,⁵³ F. Kress,⁵⁸ S. Kretschmar,¹² P. Krokovny,^{41,e} W. Krupa,³³
 W. Krzemien,³⁴ W. Kucewicz,^{32,o} M. Kucharczyk,³² V. Kudryavtsev,^{41,e} G. J. Kunde,⁶⁴ A. K. Kuonen,⁴⁶ T. Kvaratskheliya,³⁷
 D. Lacarrere,⁴⁵ G. Lafferty,⁵⁹ A. Lai,²⁵ D. Lancierini,⁴⁷ G. Lanfranchi,²¹ C. Langenbruch,¹² T. Latham,⁵³ C. Lazzeroni,⁵⁰
 R. Le Gac,⁹ R. Lefèvre,⁸ A. Leflat,³⁸ F. Lemaître,⁴⁵ O. Leroy,⁹ T. Lesiak,³² B. Leverington,¹⁵ H. Li,⁶⁸ P.-R. Li,^{4,p} X. Li,⁶⁴
 Y. Li,⁵ Z. Li,⁶⁵ X. Liang,⁶⁵ T. Likhomanenko,⁷³ R. Lindner,⁴⁵ F. Lionetto,⁴⁷ V. Lisovskyi,¹⁰ G. Liu,⁶⁸ X. Liu,³ D. Loh,⁵³
 A. Loi,²⁵ J. Lomba Castro,⁴⁴ I. Longstaff,⁵⁶ J. H. Lopes,² G. Loustau,⁴⁷ G. H. Lovell,⁵² D. Lucchesi,^{26,q} M. Lucio Martinez,⁴⁴
 Y. Luo,³ A. Lupato,²⁶ E. Luppi,^{19,f} O. Lupton,⁵³ A. Lusiani,²⁷ X. Lyu,⁴ F. Machefert,¹⁰ F. Maciuc,³⁵ V. Macko,⁴⁶
 P. Mackowiak,¹³ S. Maddrell-Mander,⁵¹ O. Maev,^{36,45} A. Maevskiy,⁷⁵ K. Maguire,⁵⁹ D. Maisuzenko,³⁶ M. W. Majewski,³³
 S. Malde,⁶⁰ B. Malecki,⁴⁵ A. Malinin,⁷³ T. Maltsev,^{41,e} H. Malygina,¹⁵ G. Manca,^{25,r} G. Mancinelli,⁹ D. Marangotto,^{24,m}
 J. Maratas,^{8,s} J. F. Marchand,⁷ U. Marconi,¹⁸ C. Marin Benito,¹⁰ M. Marinangeli,⁴⁶ P. Marino,⁴⁶ J. Marks,¹⁵ P. J. Marshall,⁵⁷
 G. Martellotti,²⁹ L. Martinazzoli,⁴⁵ M. Martinelli,^{45,23,g} D. Martinez Santos,⁴⁴ F. Martinez Vidal,⁷⁷ A. Massafferri,¹
 M. Materok,¹² R. Matev,⁴⁵ A. Mathad,⁴⁷ Z. Mathe,⁴⁵ V. Matiunin,³⁷ C. Matteuzzi,²³ K. R. Mattioli,⁷⁸ A. Mauri,⁴⁷
 E. Maurice,^{10,b} B. Maurin,⁴⁶ M. McCann,^{58,45} A. McNab,⁵⁹ R. McNulty,¹⁶ J. V. Mead,⁵⁷ B. Meadows,⁶² C. Meaux,⁹
 N. Meinert,⁷¹ D. Melnychuk,³⁴ M. Merk,³⁰ A. Merli,^{24,m} E. Michielin,²⁶ D. A. Milanese,⁷⁰ E. Millard,⁵³ M.-N. Minard,⁷
 O. Mineev,³⁷ L. Minzoni,^{19,f} D. S. Mitzel,¹⁵ A. Mödden,¹³ A. Mogini,¹¹ R. D. Moise,⁵⁸ T. Mombächer,¹³ I. A. Monroy,⁷⁰
 S. Monteil,⁸ M. Morandin,²⁶ G. Morello,²¹ M. J. Morello,^{27,t} J. Moron,³³ A. B. Morris,⁹ R. Mountain,⁶⁵ H. Mu,³
 F. Muheim,⁵⁵ M. Mukherjee,⁶ M. Mulder,³⁰ D. Müller,⁴⁵ J. Müller,¹³ K. Müller,⁴⁷ V. Müller,¹³ C. H. Murphy,⁶⁰ D. Murray,⁵⁹
 P. Naik,⁵¹ T. Nakada,⁴⁶ R. Nandakumar,⁵⁴ A. Nandi,⁶⁰ T. Nanut,⁴⁶ I. Nasteva,² M. Needham,⁵⁵ N. Neri,^{24,m} S. Neubert,¹⁵
 N. Neufeld,⁴⁵ R. Newcombe,⁵⁸ T. D. Nguyen,⁴⁶ C. Nguyen-Mau,^{46,u} S. Nieswand,¹² R. Niet,¹³ N. Nikitin,³⁸ N. S. Nolte,⁴⁵
 A. Oblakowska-Mucha,³³ V. Obraztsov,⁴² S. Ogilvy,⁵⁶ D. P. O'Hanlon,¹⁸ R. Oldeman,^{25,r} C. J. G. Onderwater,⁷²
 J. D. Osborn,⁷⁸ A. Ossowska,³² J. M. Otalora Goicochea,² T. Ovsianikova,³⁷ P. Owen,⁴⁷ A. Oyanguren,⁷⁷ P. R. Pais,⁴⁶
 T. Pajero,^{27,t} A. Palano,¹⁷ M. Palutan,²¹ G. Panshin,⁷⁶ A. Papanestis,⁵⁴ M. Pappagallo,⁵⁵ L. L. Pappalardo,^{19,f} W. Parker,⁶³
 C. Parkes,^{59,45} G. Passaleva,^{20,45} A. Pastore,¹⁷ M. Patel,⁵⁸ C. Patrignani,^{18,c} A. Pearce,⁴⁵ A. Pellegrino,³⁰ G. Penso,²⁹
 M. Pepe Altarelli,⁴⁵ S. Perazzini,¹⁸ D. Pereima,³⁷ P. Perret,⁸ L. Pescatore,⁴⁶ K. Petridis,⁵¹ A. Petrolini,^{22,l} A. Petrov,⁷³
 S. Petrucci,⁵⁵ M. Petruzzio,^{24,m} B. Pietrzyk,⁷ G. Pietrzyk,⁴⁶ M. Pikies,³² M. Pili,⁶⁰ D. Pinci,²⁹ J. Pinzino,⁴⁵ F. Pisani,⁴⁵
 A. Piucci,¹⁵ V. Placinta,³⁵ S. Playfer,⁵⁵ J. Plews,⁵⁰ M. Plo Casasus,⁴⁴ F. Polci,¹¹ M. Poli Lener,²¹ M. Poliakov,⁶⁵
 A. Poluektov,⁹ N. Polukhina,^{74,v} I. Polyakov,⁶⁵ E. Polcarpo,² G. J. Pomery,⁵¹ S. Ponce,⁴⁵ A. Popov,⁴² D. Popov,⁵⁰
 S. Poslavskii,⁴² K. Prasanth,³² E. Price,⁵¹ C. Prouve,⁴⁴ V. Pugatch,⁴⁹ A. Puig Navarro,⁴⁷ H. Pullen,⁶⁰ G. Punzi,^{27,i} W. Qian,⁴
 J. Qin,⁴ R. Quagliani,¹¹ B. Quintana,⁸ N. V. Raab,¹⁶ B. Rachwal,³³ J. H. Rademacker,⁵¹ M. Rama,²⁷ M. Ramos Pernas,⁴⁴
 M. S. Rangel,² F. Ratnikov,^{40,75} G. Raven,³¹ M. Ravonel Salzgeber,⁴⁵ M. Reboud,⁷ F. Redi,⁴⁶ S. Reichert,¹³ F. Reiss,¹¹
 C. Remon Alepuz,⁷⁷ Z. Ren,³ V. Renaudin,⁶⁰ S. Ricciardi,⁵⁴ S. Richards,⁵¹ K. Rinnert,⁵⁷ P. Robbe,¹⁰ A. Robert,¹¹
 A. B. Rodrigues,⁴⁶ E. Rodrigues,⁶² J. A. Rodriguez Lopez,⁷⁰ M. Roehrken,⁴⁵ S. Roiser,⁴⁵ A. Rollings,⁶⁰ V. Romanovskiy,⁴²
 A. Romero Vidal,⁴⁴ J. D. Roth,⁷⁸ M. Rotondo,²¹ M. S. Rudolph,⁶⁵ T. Ruf,⁴⁵ J. Ruiz Vidal,⁷⁷ J. J. Saborido Silva,⁴⁴
 N. Sagidova,³⁶ B. Saitta,^{25,r} V. Salustino Guimaraes,⁶⁷ C. Sanchez Gras,³⁰ C. Sanchez Mayordomo,⁷⁷ B. Sanmartin Sedes,⁴⁴
 R. Santacesaria,²⁹ C. Santamarina Rios,⁴⁴ M. Santimaria,^{21,45} E. Santovetti,^{28,w} G. Sarpis,⁵⁹ A. Sarti,^{21,x} C. Satriano,^{29,y}
 A. Satta,²⁸ M. Saur,⁴ D. Savrina,^{37,38} S. Schael,¹² M. Schellenberg,¹³ M. Schiller,⁵⁶ H. Schindler,⁴⁵ M. Schmelling,¹⁴
 T. Schmelzer,¹³ B. Schmidt,⁴⁵ O. Schneider,⁴⁶ A. Schopper,⁴⁵ H. F. Schreiner,⁶² M. Schubiger,³⁰ S. Schulte,⁴⁶
 M. H. Schune,¹⁰ R. Schwemmer,⁴⁵ B. Sciascia,²¹ A. Sciubba,^{29,x} A. Semennikov,³⁷ E. S. Sepulveda,¹¹ A. Sergi,^{50,45}
 N. Serra,⁴⁷ J. Serrano,⁹ L. Sestini,²⁶ A. Seuthe,¹³ P. Seyfert,⁴⁵ M. Shapkin,⁴² T. Shears,⁵⁷ L. Shekhtman,^{41,e} V. Shevchenko,⁷³
 E. Shmanin,⁷⁴ B. G. Siddi,¹⁹ R. Silva Coutinho,⁴⁷ L. Silva de Oliveira,² G. Simi,^{26,q} S. Simone,^{17,j} I. Skiba,¹⁹ N. Skidmore,¹⁵
 T. Skwarnicki,⁶⁵ M. W. Slater,⁵⁰ J. G. Smeaton,⁵² E. Smith,¹² I. T. Smith,⁵⁵ M. Smith,⁵⁸ M. Soares,¹⁸ L. Soares Lavoura,¹

M. D. Sokoloff,⁶² F. J. P. Soler,⁵⁶ B. Souza De Paula,² B. Spaan,¹³ E. Spadaro Norella,^{24,m} P. Spradlin,⁵⁶ F. Stagni,⁴⁵
 M. Stahl,¹⁵ S. Stahl,⁴⁵ P. Stefko,⁴⁶ S. Stefkova,⁵⁸ O. Steinkamp,⁴⁷ S. Stemmle,¹⁵ O. Stenyakin,⁴² M. Stepanova,³⁶
 H. Stevens,¹³ A. Stocchi,¹⁰ S. Stone,⁶⁵ S. Stracka,²⁷ M. E. Stramaglia,⁴⁶ M. Straticiuc,³⁵ U. Straumann,⁴⁷ S. Strovov,⁷⁶
 J. Sun,³ L. Sun,⁶⁹ Y. Sun,⁶³ K. Swientek,³³ A. Szabelski,³⁴ T. Szumlak,³³ M. Szymanski,⁴ Z. Tang,³ T. Tekampe,¹³
 G. Tellarini,¹⁹ F. Teubert,⁴⁵ E. Thomas,⁴⁵ M. J. Tilley,⁵⁸ V. Tisserand,⁸ S. T'Jampens,⁷ M. Tobin,⁵ S. Tolk,⁴⁵ L. Tomassetti,^{19,f}
 D. Tonelli,²⁷ D. Y. Tou,¹¹ E. Tournefier,⁷ M. Traill,⁵⁶ M. T. Tran,⁴⁶ A. Trisovic,⁵² A. Tsaregorodtsev,⁹ G. Tuci,^{27,45,i}
 A. Tully,⁵² N. Tuning,³⁰ A. Ukleja,³⁴ A. Usachov,¹⁰ A. Ustyuzhanin,^{40,75} U. Uwer,¹⁵ A. Vagner,⁷⁶ V. Vagnoni,¹⁸ A. Valassi,⁴⁵
 S. Valat,⁴⁵ G. Valenti,¹⁸ M. van Beuzekom,³⁰ H. Van Hecke,⁶⁴ E. van Herwijnen,⁴⁵ C. B. Van Hulse,¹⁶ J. van Tilburg,³⁰
 M. van Veghel,³⁰ R. Vazquez Gomez,⁴⁵ P. Vazquez Regueiro,⁴⁴ C. Vázquez Sierra,³⁰ S. Vecchi,¹⁹ J. J. Velthuis,⁵¹
 M. Veltri,^{20,z} A. Venkateswaran,⁶⁵ M. Vernet,⁸ M. Veronesi,³⁰ M. Vesterinen,⁵³ J. V. Viana Barbosa,⁴⁵ D. Vieira,⁴
 M. Vieites Diaz,⁴⁴ H. Viemann,⁷¹ X. Vilasis-Cardona,^{43,h} A. Vitkovskiy,³⁰ M. Vitti,⁵² V. Volkov,³⁸ A. Vollhardt,⁴⁷
 D. Vom Bruch,¹¹ B. Voneki,⁴⁵ A. Vorobyev,³⁶ V. Vorobyev,^{41,e} N. Voropaev,³⁶ R. Waldi,⁷¹ J. Walsh,²⁷ J. Wang,³ J. Wang,⁵
 M. Wang,³ Y. Wang,⁶ Z. Wang,⁴⁷ D. R. Ward,⁵² H. M. Wark,⁵⁷ N. K. Watson,⁵⁰ D. Websdale,⁵⁸ A. Weiden,⁴⁷ C. Weissner,⁶¹
 M. Whitehead,¹² G. Wilkinson,⁶⁰ M. Wilkinson,⁶⁵ I. Williams,⁵² M. Williams,⁶¹ M. R. J. Williams,⁵⁹ T. Williams,⁵⁰
 F. F. Wilson,⁵⁴ M. Winn,¹⁰ W. Wislicki,³⁴ M. Witek,³² G. Wormser,¹⁰ S. A. Wotton,⁵² K. Wyllie,⁴⁵ Z. Xiang,⁴ D. Xiao,⁶
 Y. Xie,⁶ H. Xing,⁶⁸ A. Xu,³ L. Xu,³ M. Xu,⁶ Q. Xu,⁴ Z. Xu,⁷ Z. Xu,³ Z. Yang,³ Z. Yang,⁶³ Y. Yao,⁶⁵ L. E. Yeomans,⁵⁷ H. Yin,⁶
 J. Yu,^{6,aa} X. Yuan,⁶⁵ O. Yushchenko,⁴² K. A. Zarebski,⁵⁰ M. Zavertyaev,^{14,v} M. Zeng,³ D. Zhang,⁶ L. Zhang,³ S. Zhang,³
 W. C. Zhang,^{3,bb} Y. Zhang,⁴⁵ A. Zhelezov,¹⁵ Y. Zheng,⁴ Y. Zhou,⁴ X. Zhu,³
 V. Zhukov,^{12,38} J. B. Zonneveld,⁵⁵ and S. Zucchelli^{18,c}

(LHCb Collaboration)

¹*Centro Brasileiro de Pesquisas Físicas (CBPF), Rio de Janeiro, Brazil*

²*Universidade Federal do Rio de Janeiro (UFRJ), Rio de Janeiro, Brazil*

³*Center for High Energy Physics, Tsinghua University, Beijing, China*

⁴*University of Chinese Academy of Sciences, Beijing, China*

⁵*Institute Of High Energy Physics (ihep), Beijing, China*

⁶*Institute of Particle Physics, Central China Normal University, Wuhan, Hubei, China*

⁷*Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS, IN2P3-LAPP, Annecy, France*

⁸*Université Clermont Auvergne, CNRS/IN2P3, LPC, Clermont-Ferrand, France*

⁹*Aix Marseille Univ, CNRS/IN2P3, CPPM, Marseille, France*

¹⁰*LAL, Univ. Paris-Sud, CNRS/IN2P3, Université Paris-Saclay, Orsay, France*

¹¹*LPNHE, Sorbonne Université, Paris Diderot Sorbonne Paris Cité, CNRS/IN2P3, Paris, France*

¹²*I. Physikalisches Institut, RWTH Aachen University, Aachen, Germany*

¹³*Fakultät Physik, Technische Universität Dortmund, Dortmund, Germany*

¹⁴*Max-Planck-Institut für Kernphysik (MPIK), Heidelberg, Germany*

¹⁵*Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg, Heidelberg, Germany*

¹⁶*School of Physics, University College Dublin, Dublin, Ireland*

¹⁷*INFN Sezione di Bari, Bari, Italy*

¹⁸*INFN Sezione di Bologna, Bologna, Italy*

¹⁹*INFN Sezione di Ferrara, Ferrara, Italy*

²⁰*INFN Sezione di Firenze, Firenze, Italy*

²¹*INFN Laboratori Nazionali di Frascati, Frascati, Italy*

²²*INFN Sezione di Genova, Genova, Italy*

²³*INFN Sezione di Milano-Bicocca, Milano, Italy*

²⁴*INFN Sezione di Milano, Milano, Italy*

²⁵*INFN Sezione di Cagliari, Monserrato, Italy*

²⁶*INFN Sezione di Padova, Padova, Italy*

²⁷*INFN Sezione di Pisa, Pisa, Italy*

²⁸*INFN Sezione di Roma Tor Vergata, Roma, Italy*

²⁹*INFN Sezione di Roma La Sapienza, Roma, Italy*

³⁰*Nikhef National Institute for Subatomic Physics, Amsterdam, Netherlands*

³¹*Nikhef National Institute for Subatomic Physics and VU University Amsterdam, Amsterdam, Netherlands*

³²*Henryk Niewodniczanski Institute of Nuclear Physics Polish Academy of Sciences, Kraków, Poland*

- ³³AGH—University of Science and Technology, Faculty of Physics and Applied Computer Science, Kraków, Poland
- ³⁴National Center for Nuclear Research (NCBJ), Warsaw, Poland
- ³⁵Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania
- ³⁶Petersburg Nuclear Physics Institute NRC Kurchatov Institute (PNPI NRC KI), Gatchina, Russia
- ³⁷Institute of Theoretical and Experimental Physics NRC Kurchatov Institute (ITEP NRC KI), Moscow, Russia
- ³⁸Institute of Nuclear Physics, Moscow State University (SINP MSU), Moscow, Russia
- ³⁹Institute for Nuclear Research of the Russian Academy of Sciences (INR RAS), Moscow, Russia
- ⁴⁰Yandex School of Data Analysis, Moscow, Russia
- ⁴¹Budker Institute of Nuclear Physics (SB RAS), Novosibirsk, Russia
- ⁴²Institute for High Energy Physics NRC Kurchatov Institute (IHEP NRC KI), Protvino, Russia, Protvino, Russia
- ⁴³ICCUB, Universitat de Barcelona, Barcelona, Spain
- ⁴⁴Instituto Galego de Física de Altas Enerxías (IGFAE), Universidade de Santiago de Compostela, Santiago de Compostela, Spain
- ⁴⁵European Organization for Nuclear Research (CERN), Geneva, Switzerland
- ⁴⁶Institute of Physics, Ecole Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland
- ⁴⁷Physik-Institut, Universität Zürich, Zürich, Switzerland
- ⁴⁸NSC Kharkiv Institute of Physics and Technology (NSC KIPT), Kharkiv, Ukraine
- ⁴⁹Institute for Nuclear Research of the National Academy of Sciences (KINR), Kyiv, Ukraine
- ⁵⁰University of Birmingham, Birmingham, United Kingdom
- ⁵¹H.H. Wills Physics Laboratory, University of Bristol, Bristol, United Kingdom
- ⁵²Cavendish Laboratory, University of Cambridge, Cambridge, United Kingdom
- ⁵³Department of Physics, University of Warwick, Coventry, United Kingdom
- ⁵⁴STFC Rutherford Appleton Laboratory, Didcot, United Kingdom
- ⁵⁵School of Physics and Astronomy, University of Edinburgh, Edinburgh, United Kingdom
- ⁵⁶School of Physics and Astronomy, University of Glasgow, Glasgow, United Kingdom
- ⁵⁷Oliver Lodge Laboratory, University of Liverpool, Liverpool, United Kingdom
- ⁵⁸Imperial College London, London, United Kingdom
- ⁵⁹School of Physics and Astronomy, University of Manchester, Manchester, United Kingdom
- ⁶⁰Department of Physics, University of Oxford, Oxford, United Kingdom
- ⁶¹Massachusetts Institute of Technology, Cambridge, Massachusetts, USA
- ⁶²University of Cincinnati, Cincinnati, Ohio, USA
- ⁶³University of Maryland, College Park, Maryland, USA
- ⁶⁴Los Alamos National Laboratory (LANL), Los Alamos, USA
- ⁶⁵Syracuse University, Syracuse, New York, USA
- ⁶⁶Laboratory of Mathematical and Subatomic Physics, Constantine, Algeria
[associated with Universidade Federal do Rio de Janeiro (UFRJ), Rio de Janeiro, Brazil]
- ⁶⁷Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio), Rio de Janeiro, Brazil
[associated with Universidade Federal do Rio de Janeiro (UFRJ), Rio de Janeiro, Brazil]
- ⁶⁸South China Normal University, Guangzhou, China
(associated with Center for High Energy Physics,
Tsinghua University, Beijing, China)
- ⁶⁹School of Physics and Technology, Wuhan University, Wuhan, China
(associated with Center for High Energy Physics, Tsinghua University, Beijing, China)
- ⁷⁰Departamento de Física, Universidad Nacional de Colombia, Bogota, Colombia
(associated with LPNHE, Sorbonne Université, Paris Diderot Sorbonne Paris Cité,
CNRS/IN2P3, Paris, France)
- ⁷¹Institut für Physik, Universität Rostock, Rostock, Germany
(associated with Physikalisches Institut,
Ruprecht-Karls-Universität Heidelberg,
Heidelberg, Germany)
- ⁷²Van Swinderen Institute, University of Groningen, Groningen, Netherlands
(associated with Nikhef National Institute for Subatomic Physics,
Amsterdam, Netherlands)
- ⁷³National Research Centre Kurchatov Institute, Moscow, Russia
[associated with Institute of Theoretical and Experimental Physics NRC Kurchatov Institute (ITEP NRC KI),
Moscow, Russia]
- ⁷⁴National University of Science and Technology “MISIS”, Moscow, Russia
[associated with Institute of Theoretical
and Experimental Physics NRC Kurchatov Institute (ITEP NRC KI), Moscow, Russia]
- ⁷⁵National Research University Higher School of Economics, Moscow, Russia
(associated with Yandex School of Data Analysis, Moscow, Russia)

⁷⁶*National Research Tomsk Polytechnic University, Tomsk, Russia*

[associated with Institute of Theoretical and

Experimental Physics NRC Kurchatov Institute (ITEP NRC KI), Moscow, Russia]

⁷⁷*Instituto de Física Corpuscular, Centro Mixto Universidad de Valencia—CSIC, Valencia, Spain*

(associated with ICCUB, Universitat de Barcelona, Barcelona, Spain)

⁷⁸*University of Michigan, Ann Arbor, USA (associated with Syracuse University, Syracuse, New York, USA)*

^aDeceased.

^bAlso at Laboratoire Leprince-Ringuet, Palaiseau, France.

^cAlso at Università di Bologna, Bologna, Italy.

^dAlso at Università di Modena e Reggio Emilia, Modena, Italy.

^eAlso at Novosibirsk State University, Novosibirsk, Russia.

^fAlso at Università di Ferrara, Ferrara, Italy.

^gAlso at Università di Milano Bicocca, Milano, Italy.

^hAlso at LIFAELS, La Salle, Universitat Ramon Llull, Barcelona, Spain.

ⁱAlso at Università di Pisa, Pisa, Italy.

^jAlso at Università di Bari, Bari, Italy.

^kAlso at Sezione INFN di Trieste, Trieste, Italy.

^lAlso at Università di Genova, Genova, Italy.

^mAlso at Università degli Studi di Milano, Milano, Italy.

ⁿAlso at Universidade Federal do Triângulo Mineiro (UFTM), Uberaba-MG, Brazil.

^oAlso at AGH—University of Science and Technology, Faculty of Computer Science, Electronics and Telecommunications, Kraków, Poland.

^pAlso at Lanzhou University, Lanzhou, China.

^qAlso at Università di Padova, Padova, Italy.

^rAlso at Università di Cagliari, Cagliari, Italy.

^sAlso at MSU—Iligan Institute of Technology (MSU-IIT), Iligan, Philippines.

^tAlso at Scuola Normale Superiore, Pisa, Italy.

^uAlso at Hanoi University of Science, Hanoi, Vietnam.

^vAlso at P.N. Lebedev Physical Institute, Russian Academy of Science (LPI RAS), Moscow, Russia.

^wAlso at Università di Roma Tor Vergata, Roma, Italy.

^xAlso at Università di Roma La Sapienza, Roma, Italy.

^yAlso at Università della Basilicata, Potenza, Italy.

^zAlso at Università di Urbino, Urbino, Italy.

^{aa}Also at Physics and Micro Electronic College, Hunan University, Changsha City, China.

^{bb}Also at School of Physics and Information Technology, Shaanxi Normal University (SNNU), Xi'an, China.